Problem Set #2

1. The earth formed by accretion of matter over a period of time, driven by gravitational attraction. We will solve a (very) simplified version of the problem to determine how hot the proto-Earth was when it reached its full size. We will assume point masses for everything, no radiation of heat energy, and no loss of mass through ejecta.

   a. Assume a particle moves inward toward the proto-Earth, which has radius $a$ and uniform density $\rho$. Assuming no atmospheric or other drag forces, and assuming the particle was at rest initially, calculate its velocity on impact. (Hint: Assume that kinetic + potential energy = 0, so that at impact the kinetic energy = -$U$).

   b. Calculate the total energy of accretion, assuming that no heat is radiated, and no material is ejected by impacts.

   c. Calculate the increase in temperature caused by accretion, assuming

      \[ \Delta T = \frac{(\text{energy/mass})}{(\text{energy/mass} \cdot ^\circ C)} \], where \((\text{energy/mass} \cdot ^\circ C) = 840 \, \text{J kg}^{-1} \, ^\circ \text{C}^{-1}\)

2. Calculate the potential and gravitational field inside and outside a thin spherical shell of radius $a$, and uniform surface density $\sigma$ (surface density $\sigma$ in units of mass/length$^2$).

3. Calculate the potential and gravitational field everywhere for a thick-walled shell (inner radius $a_1$ and outer radius $a_2$). Sketch the potential and gravitational field as a function of radius.


5. Do problem 5-7 in Turcotte and Schubert.


7. Do Problem 5-11 in Turcotte and Schubert.