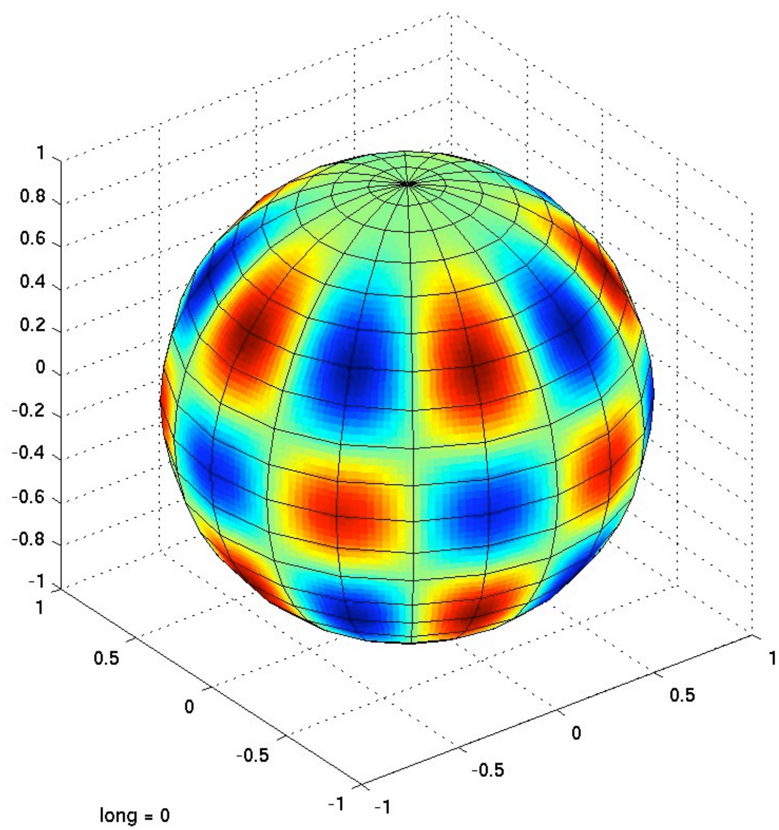
Spherical Harmonics

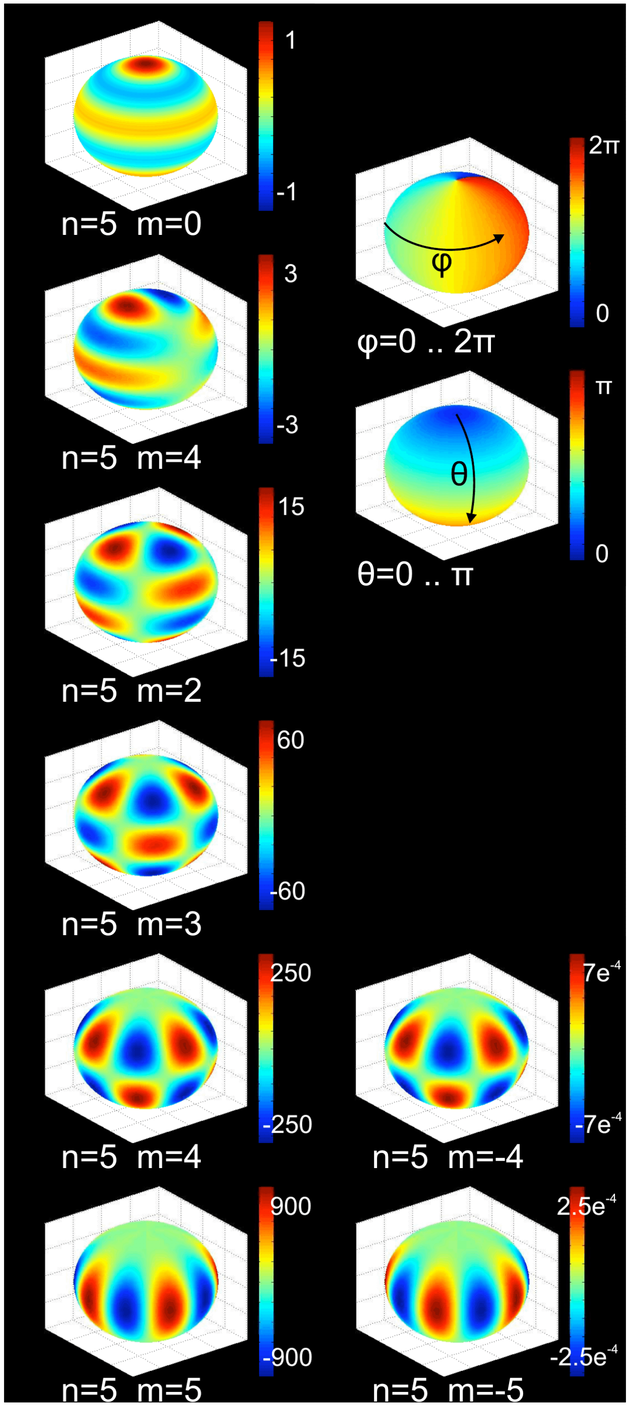


Spherical Harmonics

- One common notation for the gravitational potential is:

$$U = \frac{GM}{r} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \left(\frac{R_e}{r} \right)^n \left(C_{nm} \cos m\varphi + S_{nm} \sin m\varphi \right) P_{nm}(\cos\theta)$$

  - θ is co-latitude (0 at pole, $\pi/2$ at Equator)
  - ϕ is longitude, R_e radius of earth
- P_{nm} are the associated Legendre polynomials, derived from the Legendre polynomials P_n. These are a set of orthogonal polynomials.
- The harmonics are orthogonal, defined by integration over the surface of a sphere.
- All spherical harmonics solve Laplace's equation



Example Harmonics

- Slightly different normalization conventions are used in different fields.
  - You have to be sure to use the normalization coefficients and orthogonality relationship for the same set
- Geomagnetism: Schmidt
- Gravity/Geodesy: Fully normalized

Spherical Harmonics

- In geodesy or global gravity field studies, a slightly different notation is common:

$$Y_{inm} = P_{nm}(\cos\theta)\begin{Bmatrix} \cos m\varphi & i=1 \\ \sin m\varphi & i=2 \end{Bmatrix}$$

Normalization is implicit!

$$C_{inm} = \begin{Bmatrix} C_{nm} & i=1 \\ S_{nm} & i=2 \end{Bmatrix}$$

- So $$U = \frac{GM}{r}\sum_{i=1}^{2}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R_e}{r}\right)^n C_{inm}Y_{inm}$$
- There are other slightly different notations.
- Orthogonality: $$\iint_{sphere} Y_{inm}Y_{jpq} = \begin{Bmatrix} \dfrac{4\pi}{\Pi_{nm}^2} & i=j, n=p, m=q \\ 0 & otherwise \end{Bmatrix}$$

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